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Improved Constraints on $B \to \pi \ell \bar{\nu}_\ell$ Form Factors

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ABSTRACT

The behavior of the $B \to \pi \ell \bar{\nu}_\ell$ form factors in the entire physical range is examined in a model independent way. Unitarity bounds are further constrained by the lattice results (APE). From this analysis, we obtain $f^+(0) = 0.38 \pm .03$ if the B^*-pole dominance behavior is assumed. However, to get the information on the behavior of the form factor f^+, QSR results are included. We see the deviations from the pole dominance due to the contributions of the higher singularities which are to be treated with more precise lattice data.

1 Introduction

In this paper we would like to examine the $B \to \pi \ell \bar{\nu}_\ell$ form-factors in the entire physical region in a model independent way. This is, of course, very important for the forecoming B-factories' experiments which will provide us a way to extract the precise value of V_{ub}. This CKM- matrix element plays the crucial role in our understanding of the mechanism of CP violation. V_{ub} is mainly determined from the end-point of the lepton spectrum in semileptonic B-decays. Unfortunately, the theory of the end-point region of the lepton spectrum in inclusive $B \to X_u \ell \bar{\nu}_\ell$ decays is very complicated and suffers from large uncertainties. Hence, we explore exclusive decay modes which are also easier for experimentalists. The natural candidate is $B \to \pi \ell \bar{\nu}_\ell$. However, the determination of V_{ub} depends on the knowledge of the physics at large distances, i.e. non-perturbative QCD. This problem has proved to be notoriously difficult. We cannot rely on the heavy quark symmetry (HQS) to reduce number of the form factors or to fix their normalisationas at q^2_{max}. In heavy - heavy transition, heavy quark symmetry helps to calculate the normalisation at the zero-recoil point with small and controlled theoretical errors. This is the crucial point for the model independent determination of $|V_{cb}|$. For the case of the light resulting particle, we have no such a guidance. The knowledge of the form-factor at several points does not mean the knowledge of its functional behavior. Existing experimental values ($CLEO$) [11] for the branching ratio are model dependent (WBS [10] and Isgur-Wise [9] models):

$$Br(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell) = \begin{cases} (1.63 \pm 0.46 \pm 0.34) 10^{-4} & WSB \\ (1.34 \pm 0.35 \pm 0.28) 10^{-4} & ISGW \end{cases} \tag{1}$$

The problems with these models are related to the fact that they are always not relativistic in some aspects and they do not provide real predictions for the form factors. Small recoil behavior of the form factor was also calculated by combined HQS and the chiral perturbation theory [12]. This approach is model independent, but it is valid only in the soft-pion limit. However, there are two methods that are rooted in the QCD from the first principles: the lattice QCD ($LQCD$) and the QCD sum rules (QSR). While the lattice can be employed to explore the region close to the zero-recoil point (i.e. near q^2_{max}), the QSR give us the value of the form-factors for small q^2. Unfortunately, the regions where these two methods apply do not overlap so that the intermediate region stays uncovered (at least not with a considerable precision). So far, in the lattice approach the values of form-factors were calculated at several points (several q^2), and then extrapolated according to an ansatz of functional dependence on q^2. With faster computers and new simulations, the extrapolations will be far more constrained. For the moment, we would like to obtain the bounds on the form factors, and in a consistent way treat the existing results in order to constrain these bounds. In a beautiful series of papers, Boyd, Grinstein and Lebed [4, 5, 6] applied an old method [8] to the heavy quark systems. The idea is to use crossing symmetry and dispersion relations in order

to relate form factors with QCD perturbative calculations in an unphysical kinematic region. To constrain the bounds obtained in this way, we shall use some well estimated LQCD-results (APE), as well as some QSR-results. The paper is organised as follows: In Sec.2 we outline the basic theoretical formalism and obtain the bounds. In Sec.3 we strengthen these bounds by using existing results and make an short analysis of the q^2 behavior of the form-factors.

2 General Formalism

For the sake of completeness, in this section we recall the main features of the method (see references [8, 5, 7, 6, 4]) The current matrix element governing the $B \to \pi \ell \bar{\nu}_\ell$ semileptonic decay is parametrized as

$$< \pi(p')|V^\mu(0)|B(p) >= (p + p' - q\frac{m_B^2 - m_\pi^2}{q^2})^\mu f^+(q^2) + q^\mu \frac{m_B^2 - m_\pi^2}{q^2} f^0(q^2) \tag{2}$$

where $V^\mu = \bar{u}\gamma^\mu b$, and the form factors $f^{+,0}(q^2)$ are functions of the momentum transfer $t \equiv q^2 = (p - p')^2$. They satisfy the kinematical constraint: $f^+(0) = f^0(0)$. Here, we assume that leptons are light so that the physical region of t accessible from this decay is $0 \le t \le (M - m)^2$. The expression for the decay rate is then:

$$\frac{d\Gamma}{dq^2}(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell) = \frac{G^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2)|f^+(q^2)|^2 \tag{3}$$

where $\lambda(t) = (t + m_B^2 - m_\pi^2)^2 - 4m_B^2 m_\pi^2$ is the usual triangular function. If we want to extract the precise value of V_{ud}, it is obvious how important is the knowledge of the functional behavior of $f^+(t)$. To derive the bounds we consider the two point function:

$$\Pi^{\mu\nu} \equiv i \int d^4x e^{iqx} < 0|T(V^\mu(x)V^{\nu\dagger}(0))|0 >= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_T(q^2) + g^{\mu\nu}\Pi_L(q^2) \tag{4}$$

In the case of QCD, the structure functions $\Pi_{T,L}(q^2)$ satisfy the once subtracted dispersion relations:

$$\chi_{T,L}(Q^2) = \frac{\partial \Pi_{T,L}(q^2)}{\partial q^2} |_{q^2=-Q^2} = \frac{1}{\pi} \int_0^\infty \frac{Im \Pi_{T,L}(t)}{(t + Q^2)^2} dt \tag{5}$$

The functions $\chi_{T,L}(Q^2)$ can be reliably calculated in perturbative QCD as long as we stay in the region far from resonances. So, it suffices to calculate them at $Q^2 = 0$ where $(m_b + m_u)\Lambda_{QCD} << (m_b + m_u)^2 + Q^2$ is satisfied. To one loop they read:

$$\chi_T(0) = \frac{1}{8\pi^2 (m_b^2 - m_u^2)^5} \left[(m_b^4 - m_u^4)(m_b^4 + m_u^4 - 8m_b^2 m_u^2) - 12m_b^4 m_u^4 \log \frac{m_b^2}{m_u^2} \right] \tag{6}$$

$$\chi_L(0) = \frac{(m_b^2 - m_u^2)(m_b^2 + m_u^2 + m_b m_u)(m_b^2 + m_u^2 - 4m_b m_u) - 6m_b^3 m_u^3 \log \frac{m_b^2}{m_u^2}}{8\pi^2 (m_b^2 + m_u^2)^3} \tag{7}$$

For a massless u-quark, it gives: $\chi_T(0) = 1.27 \cdot 10^{-2}/m_b^2$ and $\chi_L(0) = 1.90 \cdot 10^{-4}$ (from now on, we shall simply note $\chi_{T,L}$, instead of $\chi_{T,L}(0)$). Also, the $O(\alpha_s)$ corrections can be included [19]and they enhance the above expressions for less than 20%.

The absorptive parts of the spectral functions $Im\Pi_{L,T}(q^2)$ can be obtained by inserting the on-shell states between the two currents on the l.h.s. of Eq.4. taking the $|\bar{B}\pi >$ as the lowest state contributing to the absorptive amplitude. Crossing symmetry states that the matrix element is described by the same form factors, but real for $t_+ \leq t \leq \infty$. After integration over the phase-space, the longitudinal part becomes:

$$Im\Pi_L(q^2) \geq \frac{[(t-t_+)(t-t_-)]^{1/2}}{16\pi t^2} t_+ t_- |f^0(t)|^2 \theta(t - t_+) \tag{8}$$

Below the onset of the $B\pi$-continuum there is only one resonance - $B^*(1^-)$, which according to its quantum numbers contributes to $Im\Pi_T$, which reads:

$$Im\Pi_T(q^2) \geq \pi f_{B^*}^2 \delta(t - m_{B^*}^2) + \frac{[(t-t_+)(t-t_-)]^{3/2}}{48\pi t^3} |f^+(t)|^2 \theta(t - t_+) \tag{9}$$

where $t_\pm = m_B \pm m_\pi$. Thus, replacing the absorptive parts in dispersion relations, we obtain the following inequalities:

$$\begin{aligned} \chi_L &\geq \int_{t_+}^{\infty} \varphi_L(t) |f^0(t)|^2 dt, \\ \chi_T &\geq \frac{f_{B^*}^2}{m_{B^*}^4} + \int_{t_+}^{\infty} \varphi_T(t) |f^+(t)|^2 dt, \end{aligned} \tag{10}$$

where we put $t^{-2} Im\Pi_i(t) \equiv \varphi_i(t)|f_i(t)|^2\theta(t-t_+)$. After integration, the pole-contribution becomes very small $(f_{B^*}^2/m_{B^*}^4)$, and can be safely neglected. We absorb this contribution intoto χ_T, anyway. To get informations about the form factors in the physical region, we map the complex t-plane onto the unit disc $|z| \leq 1$:

$$\frac{1+z}{1-z} = \sqrt{\frac{(m_B + m_\pi)^2 - t}{4 m_B m_\pi}} \tag{11}$$

By this transformation, the region $t_- \leq t \leq t_+$ is mapped into the segment of the real axis $-1 < z \leq 0$, while the $0 \leq t \leq t_-$ is mapped into $0 \leq z < 1$. Two branches of the root (or two sides of the cut) are mapped to upper and lower semicircles of $|z| = 1$. Physically, the relevant kinematic region for the process $vacuum \to \bar{B}\pi$ now lies on the

unit circle, while the region for the semileptonic $B \to \pi \ell \bar{\nu}_\ell$ decay lies inside the unit circle, on the real axis. Generically rewritten in the z-plane, inequalities (10) are:

$$\frac{1}{2\pi i}\int_{\mathcal{C}:|z|=1}\frac{dz}{z}|\phi_i(z)f_i(z)|^2 \leq \chi_i \tag{12}$$

The functions $\phi_i(z)$ are solutions of the Dirichlet's boundary problem [7] of finding an analytic function on the unit disc. Their values are known on the circle: $|\phi_i(e^{i\theta})|^2 = \varphi_i(e^{i\theta})$, where φ_i are the transformed functions from the integrals (9) . The solution is:

$$\log|\phi(z)|^2 = \frac{1}{2\pi}\int_0^{2\pi} d\theta \frac{e^{i\theta}+z}{e^{i\theta}-z}\log\varphi(e^{i\theta}) \tag{13}$$

Explicitly, our functions are:

$$\phi_T(z) = \frac{1}{\sqrt{6\pi m_B m_\pi}}\frac{(1+z)^2}{(1-z)^{9/2}}\left(\frac{m_B+m_\pi}{2\sqrt{m_B m_\pi}}+\frac{1+z}{1-z}\right)^{-5} \tag{14}$$

$$\phi_L(z) = \frac{m_B^2-m_\pi^2}{8m_B m_\pi\sqrt{2\pi m_B m_\pi}}\frac{1+z}{(1-z)^{5/2}}\left(\frac{m_B+m_\pi}{2\sqrt{m_B m_\pi}}+\frac{1+z}{1-z}\right)^{-4} \tag{15}$$

Still, the B^*-pole ($z_{pole} = -0.2519$) is below threshold and cannot be ignored when we consider f^+. We do not know the size of the residue, but the m_{pole} is known. So, to remove the pole, we multiply $f^+(z)$ by the Blaschke factor:

$$P_*(z) = \frac{z - z_{pole}}{1 - z z^*_{pole}} \tag{16}$$

which is unimodular on the unit circle and principally does not spoil our analysis although it will slightly weaken our bounds. Now, the product $\phi_i(z)f_i(z)P_*(z)$ is analytic on the unit disc and obeys (12). It should be noted that in our case, there are no branch points below the threshold. The last step in deriving the bounds is to construct the inner product:

$$(g_1, g_2) = \int_{\mathcal{C}}\frac{dz}{2\pi i z}g_1^*(z)g_2(z). \tag{17}$$

Let us choose $g_1(z) = \phi_i(z)f_i(z)$ and $g_2(z) = (1 - z z_2^*)^{-1}$. From the positivity of the inner product, determinant of the (g_i, g_j) matrix is positive, so that from these two functions we have:

$$\begin{vmatrix} \chi_i & f_i^*(z_2)\phi_i^*(z_2) \\ f_i(z_2)\phi_i(z_2) & \frac{1}{1-|z_2|^2} \end{vmatrix} \geq 0, \tag{18}$$

$\forall z_2 \in \mathrm{Int}\mathcal{C}$
or explicitly

$$|f_i(z)|^2 \leq \frac{1}{|\phi_i(z)|^2}\frac{\chi_i}{1-|z|^2} \tag{19}$$

For the case of $f^0(z)$ these bounds are depicted on the Fig.1. Coming from general principles, the bounds obtained in this way cannot be strong.

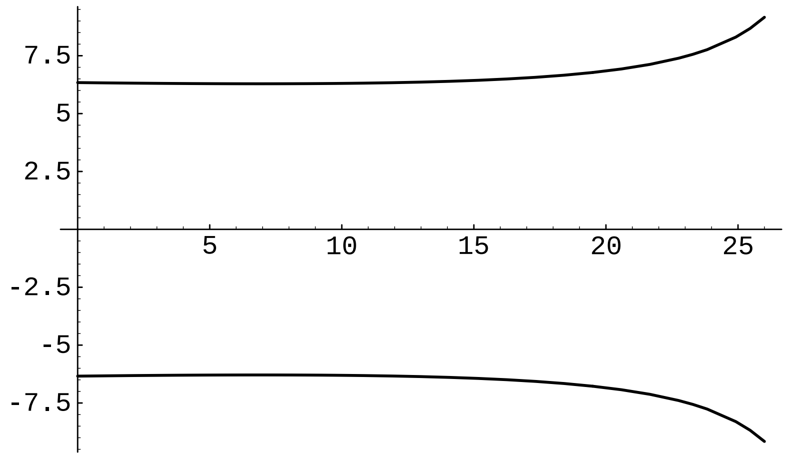

Figure 1: Unitarity bounds on the $f^0(q^2)$ form factor

However, we can impose some additional constraints.

3 Constraints and Analysis

3.1 Constraints from Lattice data

To constrain our bounds, first we shall use a set of existing lattice results (for more technical details, please see [1]). The calculations of $f^+(t)$ and $f^0(t)$ are performed on the APE machine, at $\beta = 6.0$, on a $18^3 \times 64$ lattice using the Clover action. The extrapolation to the bottom mass is done using the scaling behavior predicted by the heavy quark effective theory which is fully justified as long as the calculations are performed in the vicinity of the zero-recoil point ($q^2 \sim q^2_{max}$). The extrapolation to the light quark mass is quite smooth and unlikely to be a source of an important uncertainty within a present statistical accuracy. For the extrapolations to $q^2 = 0$, several momenta were injected, but the question of this extrapolation left opened though (the results at small q^2 suffer from large errors). For our purpose, we take three results with the best precision.

$t \equiv q^2 [GeV^2]$	17.5	20.5	24.4	26.4
f^0		.69±.10	.64±.04	.62±.04
f^+	1.06±.65	1.5±.28	2.47±.26	

Now, let us define $g_i(z) = (1 - zz_i^*)^{-1}$ ($i = 1, 2, 3$ lattice points) and again construct 5×5 matrix whose determinant is positive. Generally, for the case $i = 1, \ldots, (n-2)$, this is the $n \times n$ matrix:

$$\begin{vmatrix} \chi_i & f_i^*(z)\phi_i^*(z) & f_i^*(z_1)\phi_i^*(z_1) & \ldots & f_i^*(z_n)\phi_i^*(z_n) \\ f_i(z)\phi_i(z) & \frac{1}{1-|z|^2} & \frac{1}{1-zz_1^*} & \ldots & \frac{1}{1-zz_n^*} \\ & \ldots & \ldots & \ldots & \\ f_i(z_n)\phi_i(z_n) & \frac{1}{1-z_n z^*} & \frac{1}{1-z_n z_1^*} & \ldots & \frac{1}{1-|z_n|^2} \end{vmatrix} \geq 0 \qquad (20)$$

This is the most compact form of this expression. As it can be noticed on Fig.2, the situation for $f^0(t)$ is very much improved: while the bare unitarity bounds give $|f^0(0)| \leq 6.3$, constraints imposed by the lattice results concentrated on the opposite end ($q^2 \simeq q^2_{max}$) give $-0.7 \leq f^0(0) \leq 2.2$. Errors are included (for the upper(lower) bounds we used the upper(lower) limits from the table).

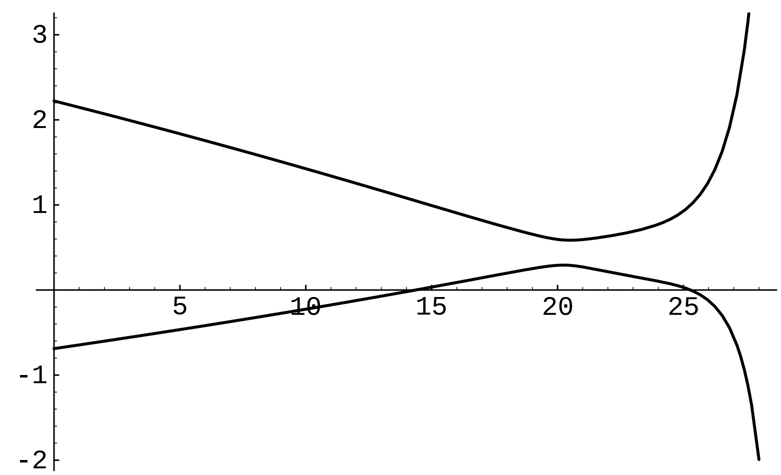

Figure 2: Constrained unitarity bounds on the $f^0(q^2)$ form factor;
3 lattice values are taken (see text)

The situation for the more interesting $f^+(t)$ is not equally good: the lattice errors are larger, the Blaschke factor affects slightly (probably because they apply equally well for any value of residue). Still, the bounds are improved. Unfortunately, the analysis shows that using the constraint $f^+(0) = f^0(0)$ does not help for further strengthening our bounds. On Fig.3, we show these bounds against the simple pole behavior (dashed

line, with $f^+(0)$ taken from Ref.[10]). We see that $-0.7 \leq f^+(0) \leq 2.7$ which means that the 'bare' allowed range is shrank by a factor of about 4.

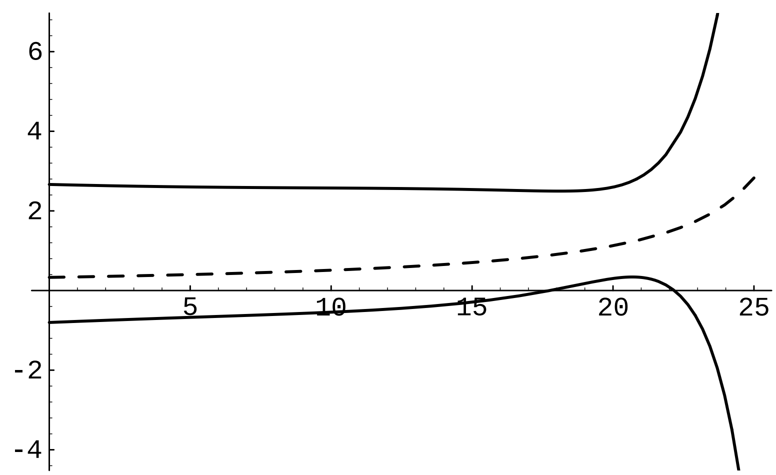

Figure 3: Constrained unitarity bounds on the $f^+(q^2)$ form factor;
3 lattice values are taken (see text)
Dashed line shows WBS model prediction

Now we can make an analysis of the methods and models applied so far in order to see if their predictions of $f^+(t)$ behavior fall within bounds in the whole region. For the moment, we want to get an information on the function $f^+(t)$.

While LQCD computations give good results around zero-recoil point, the QSR are very successefull in the oposite region. To avoid the discussion on error estimations we will take some results of the form-factors that are not far from $q^2 = 0$ point. To relate these two sets of results, we will make the fit, but with the essential physics of QCD incorporated. Again we perform the conformal mapping:

$$\frac{1+z}{1-z} = \sqrt{\frac{(m_B + m_\pi)^2 - t}{4Nm_B m_\pi}} \tag{21}$$

which is the same as the previous one for $N = 1$. Here N is to be adjusted (see below). The end points of the kinematic region are mapped as:

$$t = 0 \quad \longmapsto \quad z_{max} = \frac{m_B + m_\pi - 2\sqrt{Nm_B m_\pi}}{m_B + m_\pi + 2\sqrt{Nm_B m_\pi}} \tag{22}$$

$$t = q^2_{max} \quad \longmapsto \quad z_{min} = -\left(\frac{\sqrt{N} - 1}{\sqrt{N} + 1}\right) \tag{23}$$

We concentrate on the $f^+(t)$. Proceeding the same analysis as above, we obtain:

$$\phi_T(z) = \frac{1}{4m_B\sqrt{3\pi N m_B m_\pi}}\frac{(1+z)^2}{(1-z)^3}\left(\frac{1}{\sqrt{N}}+\frac{1+z}{1-z}\right)^{3/2}\left(\frac{m_B+m_\pi}{2\sqrt{N m_B m_\pi}}+\frac{1+z}{1-z}\right)^{-5}. \tag{24}$$

As long as $|z_{min}|$ and $|z_{max}|$ are smaller then one, $B \to \pi\ell\bar{\nu}_\ell$ decay posseses a small kinematic expansion parameter. Since both $f^+(z) \to P_*(z)f^+(z)$ and $\phi_T(z) \to \phi_T(z)/\sqrt{\chi_T}$ are analytic on the unit disc, we can Taylor expand about $z=0$:

$$f^+(z) = \frac{1}{P_*(z)\phi_T(z)}\sum_{n=0}^{\infty} a_n z^n \tag{25}$$

After the above substitutions, the transformed inequality obtained from dispersion relations gives:

$$\frac{1}{2\pi i}\int_{\mathcal{C}}\frac{dz}{z}|\phi_T(z)f^+(z)|^2 \le 1 \tag{26}$$

Eqs.(25)and(26) give:

$$\sum_{n=0}^{\infty}|a_n|^2 \le 1 \tag{27}$$

This is a very important constraint which will be used in what follows. Let us take first k-terms of this expansion:

$$f_k^+(z) = \frac{1}{P_*(z)\phi_T(z)}\sum_{n=0}^{k} a_n z^n. \tag{28}$$

k is to be chosen in such way that the truncation error is small. The expression for the maximum truncation error can be obtained using the Schwarz inequality:

$$\max|f^+(z) - f_k^+(z)| \le \max\frac{1}{P_*(z)\phi_T(z)}\sqrt{\sum_{n=k+1}^{\infty} a_n^2}\sqrt{\sum_{n=k+1}^{\infty} z^{2n}} \tag{29}$$

$$< \max\frac{1}{P_*(z)\phi_T(z)}\frac{|z^{k+1}|}{\sqrt{1-z^2}} \tag{30}$$

We could have applied this method to the previous case ($N=1$). However, in this case the physical range for $B \to \pi\ell\bar{\nu}_\ell$ ($0 \le q^2 \le 26.4 GeV^2$) would have corresponded to $0 \le z \le 0.5183$. So, in order to have small truncation error, we should have had to include so many terms in the expansion that we could not evaluate such a large number of coefficients in expansion with just few points for the fit. However, there is a way to disentangle this problem by taking the optimal value of N. In our case, we

obtain the optimal value $N = 3.45$, which gives $z_{min} = -0.300$ and $z_{max} = 0.258$. We repeat the analysis of the $f^+(t)$ constrained by the lattice results and our choice of N. To have the small truncation error (± 0.03) in the region of available lattice results, we have to take $k = 4$. Beside the three values from the table, we take the fourth one $f^+(19.75) = 1.19 \pm 0.24$ from the same simulation. The result of this analysis is very instructive. In the region around 23 GeV^2, we see that our bounds are severely constrained (Fig.4).

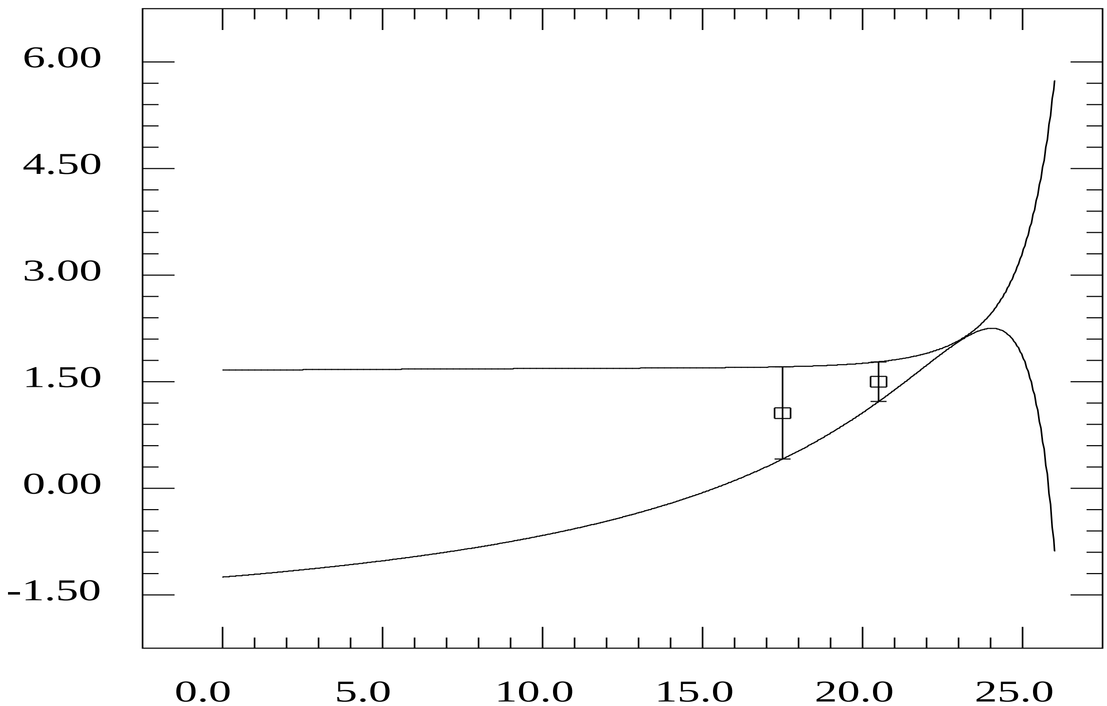

Figure 4: More constrained bounds on the $f^+(q^2)$ form factor; 3 lattice values and $N = 3.45$

It means that we can learn something about the value of the residue for this form factor. This point (23 GeV^2) is close to the zero-recoil point and is presumably dominated by the lightest state which couples to the weak current V_μ, namely the B^* ($M^2_{B^*} = 28.3 GeV^2$).

$$f^+(q^2) = \frac{F_*}{m^2_{B^*} - q^2} \tag{31}$$

(F_* stands for the residue), we obtain $F_* = (10.84 \pm .03) GeV^2$. Of course from dispersion relation for this form factor, we have also the contribution of continuum, that

we take to be negligible relative to the pole one. If pole-dominance is assumed, this value of residue gives $f^+(0) = 0.38 \pm .03$.

3.2 Inclusion of QSR data

However, our kinematic region is large and we can not that easily conclude about the value $f^+(0)$,nor the functional dependence of $f^+(t)$. To get some additional information, we shall take advantage of the knowledge of well estimated values of this form factor in the region of the small q^2. Specifically, we take the updated results of the light-cone sum rules which proved to be very stable in the range $0 \leq q^2 \leq 15\ GeV^2$. In what follows, we take only central values in both sets of results (QSR and LQCD). Again, the goal is to learn more about $f^+(t)$ behavior. For the fit, we use eight points which ensure that the truncation error is less than 0.03. Here, we recall the important constraint (27) which eventually reduce the number of coefficients contributing to (28) to four. Completed by QSR data, the set for the fit is:

$t \equiv q^2 [GeV^2]$	0	4	8	10	15	17.5	20.5	24.4
z	0.2584	0.2239	0.1829	0.1590	0.0856	0.0380	-0.0348	-0.1775
f^+	0.29	0.35	0.45	0.52	0.83	1.06	1.5	2.47

The coefficients a_n are:

a_0	a_1	a_2	a_3	a_4
$3.33 \cdot 10^{-3}$	$0.36 \cdot 10^{-3}$	$-33.29 \cdot 10^{-3}$	$-5.32 \cdot 10^{-3}$	$233.11 \cdot 10^{-3}$

On Fig.5a, we plot $f_1^+(q^2)$ (with constraint (27)) versus $f_2^+(q^2)$ (without it). On Fig.5b $f_1^+(q^2)$ versus pole behavior, and on Fig.5c $f_2^+(q^2)$ versus double pole behavior is ploted. For the pole we take (31) with the residue given below the equation, and for double pole we take. parameters from Ref.[18])

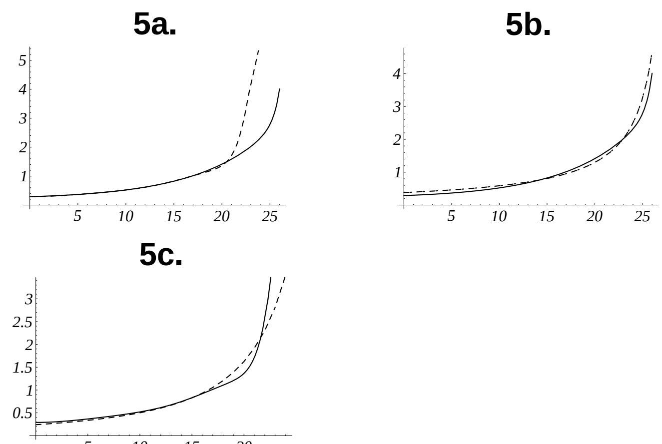

Figure 5: a) $f_1^+(q^2)$ (fit with data from the table and constraint $\sum_{n=0}^{8}|a_n|^2 \leq 1$). Dashed curve $f_2^+(q^2)$ is corresponding unconstrained fit. b) $f_1^+(q^2)$ versus $f_{pole}^+(q^2)$ (dot-dashed) c) $f_2^+(q^2)$ versus 'double-pole' behavior (dashed)

Let us briefly comment the figures. On the first plot, we see the effect of the constraint (27). It suppresses the form factor at large q^2 and constrains it to follow rather pole dominance. This is evident on the second plot. There is no compelling theoretical justification for the usage of the pole-dominance ansatz (except for the points close to the zero-recoil). However, from this analysis, we see that this behavior is strongly favored. Still, there is no convincing arguments to neglect completely the contributions of the singularities above the threshold. According to the present data, we can not estimate deviations from the simple pole behavior. Lattice data which are concentrated around q^2_{max} should be much more precise for this analysis. There is a permanent tendency in reducing all posible sources of errors (statistic and systematic) and we hope that next simulations will provide us the data for a pertinent analysis of the excited states above the threshold. As we already mentioned, for the extrapolation of the lattice data some ansätze are usually considered. These hypothesis must satisfy the kinematical constraint ($f^0(0) = f^+(0)$), as well as the scaling relations in the heavy quark limit, $M \to \infty$ ($f^0(q^2_{max} \sim M^{-1/2}$,$f^+(q^2_{max} \sim M^{1/2}$). Then, the popular ansatz is:

$$f^i(q^2) = \frac{f^i(0)}{\left(1 - \frac{q^2}{M_i^2}\right)^{n_i}} \tag{32}$$

where $n_+ = n_0 + 1$. The $UKQCD$-results suggested double pole behavior. We take their parameters and plot $f^+(q^2)$ [18] versus 'unconstrained' fit. According to our analysis,

the double pole behavior is not acceptable since the constraint (27) bends the curve and favours rather the simple-pole one.

4 Summary

There is no nonperturbative calculational method to evaluate form-factors in the whole kinematic range for the semileptonic heavy to light decays. Even more, there is no fully reliable principles which could help. Hence, all that can be done is to use a phenomenological ansatz. In this paper we wanted to attack the problem the other way around. The calculations performed in the unphysical region are related to the form factors of interest via crossing symmetry and dispersion relations. From the derived set of inequalities, using the conformal mapping, we obtained unitarity bounds on the form factors. We have shown that such bounds are not restrictive. To narrow the range of allowed values of the form factors, we used results from the simulation on the lattice (APE). Additional constraint in this method comes from the optimal choice of parameter N. As a result we get a very narrow strip of allowed values of $f^+(q^2)$ around 23.2 GeV^2. To answer the question about the q^2 behavior of the form-factor, we used the light cone QSR results. Having the essential physics incorporated, we perform a simple fit limiting ourselves to the central values predicted by the two methods. As a result, we obtained the curve which favors behavior dominated by the B^*. From $f^+(23.2\ GeV^2)$ we obtain the value of residue[2] $F_* = (10.84 \pm .03) GeV^2$ which corresponds, if the pole dominance is valid in the whole kinematic range,to $f^+(0) = 0.38 \pm .03$. Deviations from this law near q^2_{max} are present and they come from the excited B^* states which are located above the threshold. For the proper estimation of their contributions more precise lattice data are needed. The other way could be to use the $D \to \pi \ell \bar{\nu}_\ell$ form-factors and to extrapolate to $B \to \pi \ell \bar{\nu}_\ell$ armed by HQS. Then the perturbative contributions must be calculated at $Q^2 = 16 GeV^2$ and $\mathcal{O}(\alpha_f)$ corrections are large. In our analysis we used the only two nonperturbative methods so that the bounds and outlined behavior in q^2 are model independent.

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[2]this value of the residue is in a good agreement with the value of $g_{B^* B \pi}$ obtained in Ref. [3]

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